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A COBB-DOUGLAS ESTIMATION OF LABOUR PRODUCTIVITY IN THE SOUTH AFRICAN MOTOR VEHICLE MANUFACTURING INDUSTRY

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OPSOMMING

Die oogmerk van die artikel is om 'n unieke uitbreiding van die Cobb-Douglas doelmatigheidskriteria in die meting en kwantifisering van arbeidsproduktiwiteit bekend te stel. Die Suid-Afrikaanse motorindustrie is as 'n gevalle studie gebruik. Die artikel stel die hipotese dat een van die belangrike bydraende faktore tot die toenemende eenheidskoste in die industrie die oënskynlike gebrek aan arbeidsproduktiwiteit is. Die resultate van die metings is interessant en staaf die hipotese dat die kontinue lae vlak van arbeidsproduktiwiteit in die bedryf 'n sterk bydraende faktor tot stygende eenheidskoste en die oneffektiewe samestelling van die arbeid/kapitaal insetkombinasie is.

ABSTRACT

The objective of the paper is to demonstrate the use of an unique extention of the Cobb-Douglas efficiency criteria for the measurement and quantification of labour productivity. The South African motor vehicle manufacturing industry is used as a case study. This paper adopts the hypothesis that one of the more important factors contributing to the spiralling unit cost in the motor vehicle industry is the sluggish level of labour productivity. The results of the measurements are indeed interesting and it substantiates the hypothesis that the low level of labour productivity does contribute significantly to the increasing unit cost of the industry.

The South African motor vehicle manufacturing industry is regarded as an important and sensitive industry given its impact on the local job market, its effects on the balance of payments, its demand for locally manufactured raw and secondary materials and its implications for technological advancement.

Positive real price increases in the industry, as a result of the inefficient level of labour productivity, can no longer be accepted considering the imminent partial elimination of the protective tariff structure applicable to the industry. The future survival and competitiveness of the industry is critically dependent on a higher level of cost efficiency and in particular a more productive labour input.

It is therefore important to measure and quantify the extent of the perceived lack of labour productivity. This can be done within the cofines of the framework of the Cobb-Douglas efficiency criteria. The findings should help the industry, labour unions and other interests to grasp the full implications of the low labour productivity in the industry.

There are basically three applications of the Cobb-Douglas efficiency criteria that can be used to measure and quantify the level of labour productivity in the industry. These are the optimum efficiency criterium, the optimum utilisation of the total cost outlay and the optimum factor allocation warranted by the demand for vehicles.

THE OPTIMUM EFFICIENCY CRITERIUM (Ψ)

The theoretically purist and most widely used production function is the Cobb-Douglas function which states the relationship of labour (L) and capital (K) to output (Q) as: $\mathbf{Q=aK^{\alpha}L^{\beta}}$. Parameter α is the output elasticity of capital and β the output elasticity of labour. Parameter β denotes the percentage change in output as a result of a percentage change in labour input, keeping capital constant (Cobb & Douglas, 1928, pp. 139-165).

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To estimate the Cobb-Douglas production function, the function $\mathbf{Q=aK^{\alpha}L^{\beta}}$ is converted to $\mathbf{\log Q=\log a+\alpha \log K+\beta \log L}$. By means of linear regression analysis parameters a,α and β are estimated. The marginal products of labour and capital are respectively $\mathbf{MP_L=\beta(Q/L)}$ and $\mathbf{MP_K=\alpha(Q/K)}$. The optimal utilisation (cost efficiency) of a factor of production is obtained at the point where the marginal products per last Rand spent is the same for all factors: $\mathbf{(MP_L/w)=(MP_K/r)}$ where w is the unit wage cost and r the unit capital cost. The estimated production function can be used to determine whether the industry is using an optimal or sub-optimal input mix.

If $\mathbf{MP_L/MP_K<w/r}$, labour is over-utilised, indicating a decline in labour productivity. On the other hand, if $\mathbf{MP_L/MP_K>w/r}$, capital is over-utilised, thus indicating an improvement in labour productivity, ceteris paribus. The marginal rate of technical substitution (labour for capital) is: $\mathbf{MRTS=MP_L/MP_K=(\beta/\alpha).(K/L)}$. The optimum input mix (labour and capital) is such that $\mathbf{w/r=(\beta/\alpha).(K/L)}$ or $\mathbf{(\beta/\alpha).(K/L)-(w/r)=0}$. Multiplying by α yields the input efficiency criteria $\mathbf{\Psi=\beta(K/L)-\alpha(w/r)=0}$. If $\mathbf{\beta(K/L)-\alpha(w/r)<0}$ (thus $\Psi<0$) the industry is experiencing a decline in labour productivity since $\Psi<0$ implies that $\mathbf{(MP_L/MP_K)<(w/r)}$ (Maurice and Smithson, 1985, pp. 126-130).

The prices of the factors of production (w and r) can be calculated in various ways. The input price of labour (w) is obtained by dividing total salaries and wages by the number of employees. This yields the average annual remuneration per worker (expressed in R'000). In order to make the price of wages comparable to the price of capital, the average wages (in R'000) per annum are deflated by the producer price index (1990=100).

The input price of capital (r) is generally expressed as $r=Q_K(i+\delta)$ where Q_K is the unit acquisition cost of the capital stock. The rate of depreciation (δ) is calculated as the percentage of total capital depreciation.

The calculated real prices of labour and capital for the period 1972-1993 are listed in Table 1.

The production function for the motor vehicle manufacturing industry is estimated as $\mathbf{Q=7{,}653K^{0{,}384}L^{0{,}688}}$(Van Zyl en

Kleynhans, 1994, p. 32). This indicates that the industry is very labour intensive. The output elasticity of labour (β) implies that a ten per cent increase in labour productivity would result in a 6,88 per cent increase in output, ceteris paribus.

TABLE 1
THE REAL PRICES OF LABOUR AND CAPITAL IN THE MOTOR VEHICLE MANUFACTURING INDUSTRY FOR THE PERIOD 1972-1993

Year	Price of labour	Price of capital
1972	21,03	0,14516
1973	20,03	0,12129
1974	19,71	0,16159
1975	19,18	0,10850
1976	18,56	0,14763
1977	18,89	0,14704
1978	18,66	0,15209
1979	17,62	0,07411
1980	17,26	0,07295
1981	18,27	0,12112
1982	18,74	0,12481
1983	19,08	0,16624
1984	20,18	0,22985
1985	19,23	0,11482
1986	17,37	0,05331
1987	16,82	0,08822
1988	17,52	0,18554
1989	17,99	0,18829
1990	17,88	0,17878
1991	18,82	0,15176
1992	19,74	0,11559
1993	19,33	0,14361

As indicated in the theoretical exposition above, the optimal utilisation of the labour component is expressed by the efficiency criteria (Ψ) which is calculated by the use of the formula **$\Psi = \beta(K/L) - \alpha(w/r)$** (Van Zyl en Kleynhans, 1994, pp. 5-7).

The results for the period 1972-1993 are shown in Table 2.

TABLE 2
EFFICIENCY CRITERIUM (Ψ)

Year	Ψ	Input over-utilisation
1972	-48,11	L
1973	-55,92	L
1974	-39,67	L
1975	-61,36	L
1976	-42,77	L
1977	-44,19	L
1978	-41,52	L
1979	-85,62	L
1980	-84,26	L
1981	-50,24	L
1982	-49,44	L
1983	-35,16	L
1984	-23,92	L
1985	-54,24	L
1986	-115,52	L
1987	-62,61	L
1988	-26,45	L
1989	-25,91	L
1990	-26,68	L
1991	-35,41	L
1992	-51,78	L
1993	-36,37	L

From the table it is evident that $\Psi < 0$ for each and everyone of the 22 production years, irrespective of the business cycle phases. This is an indication of a continuous decline in the level of labour productivity. The next step is to quantify the decline in labour productivity in terms of cost wastage (in Rand).

THE OPTIMUM UTILISATION OF THE TOTAL COST OUTLAY

The optimal input ratio of labour and capital must be such that **$K/L = (\alpha w)/(\beta r)$**. The optimum allocation of the labour input can be calculated from the optimal input ratio. Thus **$L = (\beta rK)/(\alpha w)$**.

The optimum allocation of the labour input can also be derived from the isocost line for a specific cost outlay: **$C = rK + wL$ and $L = (C - rK)/w$**.

In practice it is, however, almost impossible to obtain a level of production where $\Psi = 0$. Should the calculations show no optimal input allocation, it must be determined whether the calculated Ψ is significantly different from 0. This is done by means of a t-test. The calculated t-statistic is $t = \Psi/S\Psi$. $S\Psi$ is the estimated Ψ's own standard error and the estimated variance of Ψ can be calculated as:

$$Var(\Psi) = (K/L)^2 Var(\beta) + (w/r)^2 Var(\alpha) - 2(K/L)(w/r)Cov(\alpha,\beta).$$

The estimated standard error of Ψ is:

$$S\Psi = \sqrt{Var(\Psi)}$$

The absolute t-value of Ψ is then calculated. Should and it exceed the critical t-value, it can be said that Ψ is significantly different from 0 (Maurice and Smithson, 1985, pp. 128-130).

The intensity factor and the factor demand equations are two usefull equations derived from the Cobb-Douglas function. When evaluating the optimum total cost outlay, it is important to take note of these two equations. The intensity factor is **(β/α)**.

The higher this ratio the more labour intensive the production technique (Koutsoyiannis, 1979, p. 76). When **$\Psi = 0$, $\beta(K/L) - \alpha(w/r) = 0$** thus **$L = K(\beta/\alpha)(r/w)$**. Substituting K and L in the production function the factor demand equations are derived in terms of output and relative factor prices. **$Ld = [Q/L(r/w.\beta/\alpha)^{\delta}]^{1/(\alpha+\beta)}$** (Heathfield, 1987, p. 82).

The low levels of labour productivity in the industry over the period 1972-1993 had an adverse effect on the output level. From the results (see table 3) it is evident that a better utilisation (input mix) of the cost outlay would have resulted in a higher output level.

The possible optimal output gain was calculated for every year since 1972. For example, 1993: K = R1562m; L = 70,185; w = 19,3325; r = 0,14361; $\Psi = -36{,}3719$ indicating an over-utilised labour situation (decline in productivity).

The true ratio K/L = 22,27
$(K/L)_0$ ratio = (αw)/(βr) = 75,1357
C = rK + wL
∴ C = R1,5813b
$K_0 = (\alpha C)/(r\varepsilon)$ = R3,944b
$L_0 = (\beta rK)/(\alpha w)$ = 52496 workers.
Test: K_0/L_0 = 75,1357
$Q = aK^{\alpha}L^{\beta}$
Q_{true} = R3,9493b
Qo = R4,6146b
The inefficiency output loss = R665,203m.

The inefficient labour component is calculated as L_{true} - Lo = 17689 excess workers.

Table 3 shows the possible output wasted in each year (1972-1993) and the unproductive labour that was employed at the non-optimal factor allocation and total cost outlay levels.

TABLE 3
NON-OPTIMAL UTILISATION OF THE TOTAL COST OUTLAY FOR THE PERIOD 1972-1993

Year	Output loss (R billion)	Unproductive labour (excess workers)
1972	1,13	18435
1973	1,38	20022
1974	1,13	20732
1975	1,82	23512
1976	1,52	23931
1977	1,50	22721
1978	1,38	22331
1979	2,44	24324
1980	2,50	26110
1981	1,72	26649
1982	1,73	27718
1983	1,10	23712
1984	0,68	20832
1985	1,42	23052
1986	2,58	24368
1987	1,47	21980
1988	0,84	23571
1989	0,76	22442
1990	0,72	21675
1991	0,88	21441
1992	1,12	21237
1993	0,67	17689

From the table, it is obvious that the calculated output loss as a result of the employment of unproductive labour remained relatively high over the entire period.

OPTIMAL FACTOR ALLOCATION (LABOUR AND CAPITAL) ACCORDING TO MARKET DEMAND

When an optimal factor allocation at a given labour and capital cost has been determined for a particular cost outlay, it can be used to determine the optimal factor allocation warranted by the market demand.

The optimum amount of labour in the industry required to meet market demand can be calculated by the use of the formula

$$Ld = {}^{\varepsilon}\sqrt{(Q_D K_0^{\alpha} L_0^{\beta})/(Q_0 Z^{\alpha})}$$

where Z = optimal K/L = $(\alpha w)/(\beta r)$ and $\varepsilon = \alpha + \beta$). For given factor prices it follows that the optimum labour input for a desired output/market demand can be caclulated by the use of the formula

$$L_D = {}^{\varepsilon}\sqrt{Q_D/(Z^{\alpha} L)}$$

(Van Zyl en Kleynhans, 1994, pp. 8-10)

According to spokesmen in the industry, motor vehicles manufactured are to be sold within six weeks. We therefore assume that the number of motor vehicles manufactured, are determined by market demand. Current production could, however, be maintained at a lower cost should inputs be better utilised.

The non-productive labour component per level of market demand, the possible cost gain as a result of better factor allocation and the waste per unit of production are culculated for each year over the period 1972-1993. These figures are listed in table 4.

TABLE 4
FACTOR WASTAGE ACCORDING TO MARKET DEMAND FOR THE PERIOD 1972-1993

Year	Market demand R billion	Unproductive labour	Possible cost gain with less labour employed (R million)	Waste per unit (R)
1972	2,516	30456	404	1097
1973	2,693	33878	469	1028
1974	2,864	33458	407	930
1975	2,943	41274	266	1189
1976	2,877	40769	393	1320
1977	2,622	39302	432	1548
1978	2,704	37767	390	1203
1979	2,791	45630	530	1517
1980	3,225	47882	486	1178
1981	3,758	44167	488	895
1982	4,069	45341	448	983
1983	3,841	36165	584	744
1984	3,928	29326	585	536
1985	3,677	31029	498	1117
1986	3,476	44314	514	1661
1987	3,514	40743	370	906
1988	4,3567	33728	267	631
1989	4,4341	31512	418	587
1990	4,4917	30158	539	574
1991	4,1932	31059	344	745
1992	4,0806	32120	277	960
1993	3,9493	24784	254	584

From the table, it is obvious that the industry is burdened by a significant number of non-productive labourers. These figures are disturbing when compared with tables 1-3. According to table 1 real wages remained relatively unchanged while the number of non-productive labourers had increased. This is an indication of a continuous decline in labour productivity.

CONCLUSION

The Cobb-Douglas efficiency criteria and in particular it's extentions serve as effective and useful instruments to measure and quantify the extent of a decline in labour productivity in a particular industry.

The results of the efficiency criteria measurements do indeed substantiate the view that the continuous decline in labour productivity in the motor vehicle industry is one of the more important causes of the higher unit costs. The key challenge facing all those associated with the industry is the improvement of labour productivity at a time when the overall productivity trend remains under pressure and wages and other costs have been rising at a faster rate than those of the overseas competitors.

The demands of the labour unions have probably compelled the industry to employ more labour at higher wage levels than would have been the case had management been at liberty to act more rationally. The low level of productivity per worker can probably also be attributed to the low worker ethics generally prevalent in the South African work force. The findings of this paper agree with those of the Riley report (IDC, 1993, pp. 99-102) of the Motor Industry Task Group that the South African motor vehicle industry is too labour intensive. It is obvious that should the decline in labour productivity remains unchallenged, thousands of workers will have to be retrenched in order to bring unit cost down and make the industry more competitive.

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